

AD-A258 640



AEROSPACE REPORT NO.
ATR-91(7234)-2

**Force Balance Near an X Line
Along Which $E \cdot J < 0$**

DTIC
ELECTE
DEC 14 1992
S c D

Prepared by

L. R. LYONS and D. C. PRIDMORE-BROWN
Space and Environment Technology Center
Technology Operations

15 September 1992

Prepared for

NASA HEADQUARTERS
Washington, DC 20546

Grant No. NAGW-2126

Engineering and Technology Group

92-31325



THE AEROSPACE CORPORATION

El Segundo, California

PUBLIC RELEASE IS AUTHORIZED

92 12 14 000

TECHNOLOGY OPERATIONS

The Aerospace Corporation functions as an "architect-engineer" for national security programs, specializing in advanced military space systems. The Corporation's Technology Operations supports the effective and timely development and operation of national security systems through scientific research and the application of advanced technology. Vital to the success of the Corporation is the technical staff's wide-ranging expertise and its ability to stay abreast of new technological developments and program support issues associated with rapidly evolving space systems. Contributing capabilities are provided by these individual Technology Centers:

Electronics Technology Center: Microelectronics, solid-state device physics, VLSI reliability, compound semiconductors, radiation hardening, data storage technologies, infrared detector devices and testing; electro-optics, quantum electronics, solid-state lasers, optical propagation and communications; cw and pulsed chemical laser development, optical resonators, beam control, atmospheric propagation, and laser effects and countermeasures; atomic frequency standards, applied laser spectroscopy, laser chemistry, laser optoelectronics, phase conjugation and coherent imaging, solar cell physics, battery electrochemistry, battery testing and evaluation.

Mechanics and Materials Technology Center: Evaluation and characterization of new materials: metals, alloys, ceramics, polymers and their composites, and new forms of carbon; development and analysis of thin films and deposition techniques; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; development and evaluation of hardened components; analysis and evaluation of materials at cryogenic and elevated temperatures; launch vehicle and reentry fluid mechanics, heat transfer and flight dynamics; chemical and electric propulsion; spacecraft structural mechanics, spacecraft survivability and vulnerability assessment; contamination, thermal and structural control; high temperature thermomechanics, gas kinetics and radiation; lubrication and surface phenomena.

Space and Environment Technology Center: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation; propellant chemistry, chemical dynamics, environmental chemistry, trace detection; atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiative signatures of missile plumes, and sensor out-of-field-of-view rejection.

FORCE BALANCE NEAR AN X LINE
ALONG WHICH $E \cdot J < 0$

Prepared by

L. R. Lyons and D. C. Pridmore-Brown
Space and Environment Technology Center
Technology Operations

15 September 1992

Engineering and Technology Group
THE AEROSPACE CORPORATION
El Segundo, CA 90245-4691

Prepared for

NASA HEADQUARTERS
Washington, DC 20546

Grant No. NAGW-2126

DTIC QUALITY ASSURED 2


PUBLIC RELEASE IS AUTHORIZED

Accession For	
NTIS	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

FORCE BALANCE NEAR AN X LINE
ALONG WHICH $E \cdot J < 0$

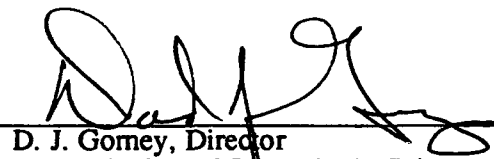
Prepared

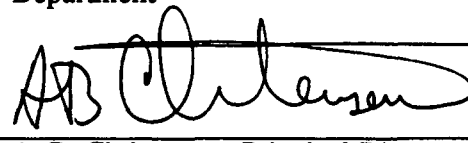

L. R. Lyons


D. C. Pridmore-Brown

Approved


J. B. Blake, Director
Space Particles and Fields Department


D. J. Gorney, Director
Atmospheric and Ionospheric Sciences
Department


A. B. Christensen, Principal Director
Space and Environment Technology
Center

NOTE

The material reproduced in this report originally appeared in *Journal of Geophysical Research*.
The ATR is published to document the work for the corporate record.

Force Balance Near an X Line Along Which $E \cdot J < 0$

L. R. LYONS AND D. C. PRIDMORE-BROWN

Space Sciences Laboratory, The Aerospace Corporation, Los Angeles

Observations suggest that plasma can occasionally be transferred from closed to open field lines in the geomagnetic tail during periods when the separatrix between open and closed field lines moves rapidly toward decreasing latitudes. Such transfer would require $E \cdot J < 0$ along the distant tail X line (E being the electric field and J the current density), and it would give important information on the physical processes that occur near an X line. The condition $E \cdot J < 0$ is not allowed in resistive reconnection theory. Collisionless reconnection theory, however, requires a balance between the electric force and a gyroviscous force at an X line but does not necessarily require that $E \cdot J > 0$. We have recently shown that gyroviscosity can provide the force required to balance the electric force along an X line having $E \cdot J > 0$. Here we extend this previous analysis of gyroviscosity to the case where $E \cdot J < 0$ along an X line. We show how the balance between gyroviscosity and the electric force along an X line is maintained for any $E \cdot J$. We conclude that observational evidence for the occasional existence of $E \cdot J < 0$ along an X line provides support for the suggestion that collisionless gyroviscosity, rather than resistivity, balances the electric force along an X line. We also evaluate conditions for which particle motion in the vicinity of an X line can give a current in the direction appropriate for $E \cdot J < 0$, and we find that there is a maximum electric field magnitude for particles to be able to carry a significant current. For parameters typical of the distant magnetotail the critical electric field magnitude was found to be ~ 0.15 mV/m, which is of the order of, though somewhat less than, the potential electric field magnitudes expected in the magnetotail. This maximum allowable field magnitude is about the same for protons as it is for electrons in the magnetotail.

1. INTRODUCTION

Solar wind flow across open (polar cap) magnetic field lines generates an electric field that extends throughout the magnetosphere. Energy and plasma are transferred to the region of closed field lines by flowing across the separatrix between open and closed field lines. The transfer to closed field lines is believed to occur primarily on the nightside of the magnetosphere. It is associated with $E \cdot J > 0$ along the magnetic X line in the distant tail [Vasyliunas, 1984], where E is the total electric field (potential plus induced) and J is the current density. Such transfer of energy and plasma across a magnetic separatrix, is commonly called "reconnection," and understanding the physical processes that occur near an X line is important for understanding the reconnection process.

The rate at which plasma is transferred across the separatrix is affected by motion of the separatrix, whose location can vary with time by more than 10° of invariant latitude Λ . Lyons *et al.* [1989] recently proposed that the nightside portion of the separatrix could occasionally move toward decreasing Λ at a faster speed than the speed at which plasma is convected toward lower Λ by magnetospheric electric fields. This would cause plasma to be transferred from closed to open field lines on the nightside, whereas the transfer normally associated with reconnection in the tail is from open to closed field lines. Such a transfer can only occur if $E \cdot J < 0$ along the tail X line and can be referred to as "reverse reconnection." The occasional existence of reverse reconnection would thus provide an important constraint on the physics near an X line. Observational evidence that $E \cdot J < 0$ is possible in the nightside magnetosphere has been provided by a recent ground-based radar study [de la Beaujardiere *et al.*, 1991]. The boundary between open and

closed magnetic field lines (as inferred from the poleward boundary of the auroral oval) was found to have moved equatorward faster than the convecting plasma for about 10–15 min during the night of January 14–15, 1989, suggesting that plasma was being transferred from closed to open field lines.

The condition $E \cdot J < 0$ at the tail X line should occur only during periods of rapid equatorial motion of the separatrix. Such motion would result from significant temporal changes in the large-scale magnetospheric magnetic field such that the induced electric field is directed opposite to that of the cross-tail current [Lyons *et al.*, 1989]. Moreover, the induced component of E would have to exceed the potential component (which is in the direction of J) in magnitude at the X line. The condition $E \cdot J > 0$ transfers particles and energy from open field lines to closed field lines in the plasma sheet. The condition $E \cdot J < 0$ would extract energy and particles from the preexisting plasma sheet distribution. Here we investigate criteria for which $E \cdot J < 0$ is feasible near an X line in a collisionless plasma. We primarily consider the phenomenon in general, but we also include its application to the Earth's magnetosphere.

It is not possible for $E \cdot J < 0$ to exist in traditional reconnection theory because it is assumed that $E = \eta J$ along an X line, where η is some sort of resistivity that must be positive. However, particle inertial effects can also balance the electric force where the magnetic field is small [Speiser, 1970]. Dungey [1988] considered reconnection from the viewpoint of particle kinetics. He found that a skewing of the electron velocity distribution should result from collisionless electron motion near an X line and that this skewing would give rise to a form of "electron viscosity" associated with the gradient of the off-diagonal elements of the pressure tensor. Such viscosity is caused by an electric field transverse to a magnetic field having an intensity that varies significantly over an electron gyroradius and has been referred to as "gyroviscosity." The potential importance of

Copyright 1992 by the American Geophysical Union.

Paper number 91JA02799.

0148-0227/92/91JA-02799\$05.00

this viscosity has been discussed by Vasyliunas [1975, 1984] and Sonnerup [1979, 1988]. Dungey proposed that this electron gyroviscosity can balance the electric force along an X line in a collisionless plasma, so that resistivity is not required.

We [Lyons and Pridmore-Brown, 1990; hereafter referred to as paper 1] evaluated terms in the electron momentum equation using a two-dimensional magnetic field model and a simplified model of particle motion based on Dungey [1988]. This evaluation showed that Dungey's basic proposal is correct and that gyroviscosity can in general balance the force from an electric field along an X line for $E \cdot J > 0$. We found the gyroviscous force to be proportional to E , so that it can balance the force from any electric field along an X line in the direction of J .

In paper 1 we calculated particle trajectories within the current sheet in the vicinity of an X line. These trajectories are clearly reversible in the sense that if the sign of the particle velocity and that of the magnetic field are both reversed, the particles will move in the opposite direction along the trajectories. While the particle motion in paper 1 was consistent with $E \cdot J > 0$, the reversed trajectories with the reversed sign of the magnetic field are consistent with $E \cdot J < 0$. This implies that collisionless particle motion can be consistent with $E \cdot J < 0$ as well as $E \cdot J > 0$ along an X line. This also implies that there is no constraint on the sign of the gyroviscous force near an X line; the proportionality between the gyroviscous force and the electric force at an X line must be independent of the sign of $E \cdot J$. Here we modify the model used in paper 1 in order to evaluate electron trajectories and the electron forces near an X line when $E \cdot J < 0$.

From the reversibility of the trajectories in paper 1, it is clearly possible to have particle motion in the direction appropriate for $E \cdot J < 0$. For electrons, which have charge $q < 0$, this requires net motion in the direction opposite to the direction of qE . Particles that are trapped on current sheet field lines and drift toward the X line provide the source for current-carrying particles near an X line having $E \cdot J < 0$. If the electric field during periods of $E \cdot J < 0$ were equal in magnitude to that during periods of $E \cdot J > 0$, and the trapped particle distributions were identical to those calculated in paper 1, then all particles in the vicinity of the X line would be able to move in the direction appropriate for $E \cdot J < 0$. However, such conditions are unlikely to occur in the Earth's magnetosphere. Particle distributions of the form calculated in paper 1 may exist very near an X line, but they are unlikely to persist after several electron bounces because of particle scattering and temporal current and field variations. By following particle trajectories, we are able to determine conditions under which particle motion can support $E \cdot J < 0$.

Note that the collisionless particle motion described here involves no turbulent energy dissipation. Thus entropy is conserved for the flow across a magnetic separatrix, so that such flow can be reversible. It is likely that there will be turbulent energy dissipation for particles trapped on field lines that cross the current sheet; however, such dissipation is not fundamental to the process of particle transfer across the magnetic separatrix. This is unlike collisionless shocks, whose solution (as distinct from soliton solutions) requires some sort of microturbulent energy dissipation [Tidman and

Krall, 1971]. Such dissipation causes entropy to increase across shocks so that they are in principle irreversible.

2. PARTICLE TRAJECTORIES

We expect reverse reconnection to occur only during periods of rapid equatorial motion of the magnetic separatrix, so that $E \cdot J < 0$ at the tail X line should only occur in association with induced electric fields. Ideally, therefore, we should model E and the magnetic field B globally as time-dependent fields and specify the motion of the separatrix from $\partial B / \partial t = -\nabla \times E$. This would be a major effort.

Before embarking on such an effort, it is worthwhile to determine conditions for which it is feasible to maintain force balance and current in a collisionless plasma having $E \cdot J < 0$ in the vicinity of an X line. Here we investigate these conditions using a simple model that elucidates the plasma forces and particle motion. Accordingly, we generalize the simplified, time-independent model of Dungey [1988] and the analysis of paper 1 to the situation where $E \cdot J < 0$.

Dungey's model for the fields in the vicinity of an X line consists of a B_x that reverses in sign across a current sheet that lies in the $z = 0$ plane and a B_z that varies linearly with x and goes through zero at the X line. Current is directed in the $+y$ direction, and E is uniform and directed parallel (or antiparallel) to the current. The model has $\nabla \times E = 0$, consistent with its time independence.

The basic motion of particles in the vicinity of a current sheet having a weak but uniform B_z was originally presented by Speiser [1965] and is described in paper 1. Particles that reach the midplane of the current sheet oscillate about the midplane and simultaneously gyrate about B_z for one-half a gyrocicle; the gyration begins with an x component of velocity $v_{x0} < 0$ (for $B_z > 0$) and $v_{y0} = 0$. This gives a current in the positive y direction, as is required to maintain the current of the current sheet. Only electrons with $v_y < 0$ remain in the current sheet. This is because electrons with $v_y > 0$ are accelerated away from the current sheet by the magnetic force due to B_x (which the electrons encounter as a result of their oscillatory motion about $z = 0$).

The effect of a uniform E_y on the particle motion is given by simply adding a component of velocity $v_x = E_y / B_z$ to the particle motion, independent of the sign of E_y . A current is maintained in the positive y direction, even if E_y is directed in the negative y direction, by a magnetic force on electrons that enter the current sheet which exceeds the electric force. It is necessary, however, that $|v_{x0}| > |E_y / B_z|$ for negative E_y ; otherwise, acceleration in the negative y direction by E_y would exceed the acceleration in the positive y direction by $v_{x0} \times B_z$. Thus there is a maximum magnitude of E_y for which electrons can contribute to a current having $E \cdot J < 0$. This maximum value in a current sheet with constant B_z is given approximately by $|E_y| \approx \langle v_{x0} \rangle B_z$, where the angle brackets denotes mean value. If E_y exceeds this magnitude, then the current near the X line will be greatly reduced by the reduction in the current-carrying particles. As the particle trajectories described below show, an analogous maximum $|E_y|$ occurs in the vicinity of an X line having $E \cdot J < 0$, even though B_z varies with x and goes through zero.

To evaluate particle trajectories near an X line, we apply, as in paper 1, two important features of the x, y component of particle motion within the current sheet mentioned above.

First, the gyration about the normal magnetic field begins with $|v_y| \ll |v_x|$ and with v_x directed toward the X line. Second, only $v_y < 0$ is allowed for negatively charged particles ($v_y > 0$ for positively charged particles). We presume that these features of the motion are unchanged by realistic spatial variations of B_z near an X line, and particle trajectories calculated by *Martin and Speiser* [1988] in the vicinity of a neutral line appear to be consistent with this presumption.

Also, as in the work by *Dungey* [1988] and paper 1, we take advantage of the decoupling that occurs between the x , y component and the z component of motion for particles as they oscillate about the $z = 0$ plane [Speiser, 1965, 1968]. We assume here that this decoupling holds as the x , y component of a particle trajectory crosses an X line. However, Speiser did not specifically include this situation or the effects of $\partial B_z / \partial x \neq 0$ in his analysis, and it would be desirable to study the validity of this assumption further.

We thus calculate only the x , y component of the particle trajectory at $z = 0$. We assume that this motion begins with $v_y = 0$ and with v_x directed toward the X line, which we take to lie along the y axis at $x = z = 0$. Particles are assumed to be ejected from the current sheet when qv_y becomes negative. We also take $\partial/\partial y = 0$, $B_y = 0$, and E uniform and in the y direction. We Taylor-expand the component of B normal to the current sheet so as to obtain $B_z = \beta x$.

Using the normalized temporal and spatial variables introduced by *Dungey* [1988] for particles of charge q and mass m :

$$x' = x/L, \quad L = |E_y m / (\beta^2 q)|^{1/3} \quad (1)$$

$$t' = t/\tau, \quad \tau = L^2 \beta / |E_y| \quad (2)$$

the equations of motion become

$$dv'_x/dt' = x' v'_y \text{ sign}(q) \quad (3)$$

$$dv'_y/dt' = (-x' v'_x \pm 1) \text{ sign}(q) \quad (4)$$

where the plus or minus sign refers to the sign of E_y , which we take to be negative in the present work. Note that E_y , m , β , and q all have a magnitude of unity in normalized units. Also, since the equations of motion are applicable to particles of any charge, we take $\text{sign}(q) = +1$. Thus our results are applicable to electrons and ions provided the sign of v'_y is changed for electrons.

We now examine particle trajectories in normalized units obtained by solving (5) and (6) using a Runge-Kutta scheme. After doing this, we will discuss what the results mean in terms of parameters appropriate for the distant geomagnetic tail. Figure 1 shows representative trajectories in the x' , y' plane in the vicinity of the X line at $x' = 0$. In each panel, trajectories are shown for particles started along the $y' = 0$ axis at initial values of x'_0 given by $x'_0 = \pm 1, \pm 1.5, \pm 2, \pm 2.5$, and ± 3 . Initial velocities v'_{x0} are the same in each panel and have magnitudes 3, 2, and 1 in Figures 1a, 1b, and 1c, respectively.

The trajectories for $|v'_{x0}| = 3$ and 2 are similar to those obtained in paper 1 for positive E_y . Here the particles are accelerated in the negative y direction by E_y (which has magnitude 1 in normalized units) as they gyrate in the positive y' direction. However, for all the initial x' values chosen, the particles move in the positive y' direction until

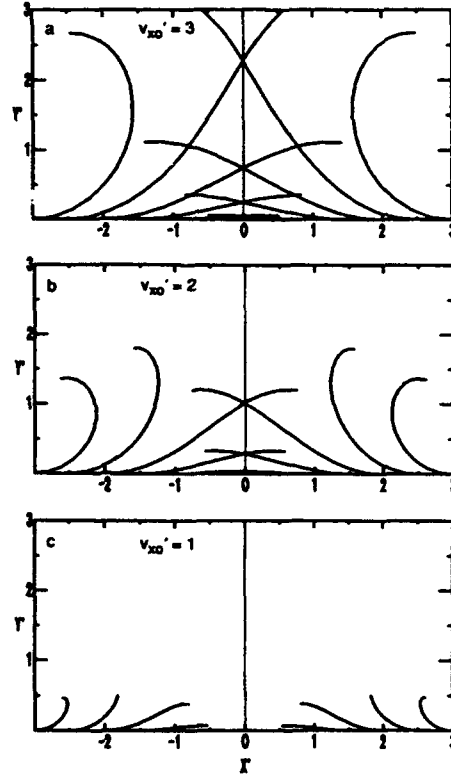


Fig. 1. Representative particle trajectories in the x' , y' plane near an X line at $x' = 0$ when $E \cdot J < 0$. Coordinates are normalized to the scale length L given by (3). The sign of y' is appropriate to a positively charged particle. For negatively charged particles the sign of y' must be changed. Trajectories are shown for particles started along the $y' = 0$ axis at initial values of x'_0 given by $x'_0 = \pm 1, \pm 1.5, \pm 2, \pm 2.5$, and ± 3 . Initial velocities v'_{x0} have magnitude (a) 3, (b) 2, and (c) 1. For $|v'_{x0}| = 1$, no trajectories reach $x' = 0$, so that no current can be carried by these particles within a region surrounding the X line.

v'_y returns to zero. (At that point, particles are ejected from the current sheet.) Some particles gyrate about B_z without crossing the X line, while those with x'_0 sufficiently close to zero cross the X line. As can be seen, there are particles carrying current in the positive y direction at all x' for these values of v'_{x0} .

For $|v'_{x0}| = 1$ we obtain significantly different results. The trajectories initiated at $|x'_0| \leq 2.5$ do not complete a half gyration about B_z , nor do they cross the X line. For these trajectories the acceleration by E_y returns v_y to zero before the particles cross the neutral line. In fact, for $|x'_0| < 1$, the initial electric field acceleration exceeds the initial magnetic field acceleration, and particles do not remain in the current sheet at all. As a result, there are no trajectories that cross the X line for $|v'_{x0}| = 1$, and no current can be carried by particles in the positive y direction within a region surrounding the X line.

Trajectories such as these show that there is a critical magnitude v'_c for the initial normalized velocity below which particles cannot carry a current having $E \cdot J < 0$ at an X line. For the model employed here, we have found via trajectory calculation and analytical approximation to the equations of

motion that $v'_c \approx 1.35$. Thus for a particle distribution incident upon the current sheet in the vicinity of an X line with a mean velocity $\langle v'_{x0} \rangle$, we have to a first approximation that $|\langle v'_{x0} \rangle|$ must exceed 1.35 for the particles to carry a significant current having $E \cdot J < 0$. (For $E \cdot J > 0$, the critical speed corresponds to a minimum final speed that particles attain after entering the current sheet at $x' = 0$.)

This critical normalized velocity can be used to estimate a maximum allowable magnitude for E_y in the direction opposite to a current at an X line. This estimate is obtained by assuming that there is a preexisting electron population along fields that cross the current sheet near the X line and that these electrons have a mean velocity $\langle v'_{x0} \rangle$. The condition that $|\langle v'_{x0} \rangle|$ exceed 1.35 then imposes a maximum allowable magnitude for E_y in the direction opposite to the current given by

$$|E_y| = |\langle v'_{x0} \rangle / 1.35|^{3/2} (\beta m_e / e)^{1/2} \quad (5)$$

Equation (5) is obtained from (1) and (2) using $v = v'/(L/\tau)$.

To evaluate (5), we assume a thermal speed of 5.9×10^6 m/s (100 eV) for electrons in the distant tail. Using this speed for $\langle v'_{x0} \rangle$ and taking $\beta = 0.3$ nT/ R_E as an order-of-magnitude estimate for the tail, we obtain $|E_y| \sim 0.15$ mV/m. This magnitude is somewhat less than typical values of the potential electric field in the tail (0.24 mV/m for a 60-kV cross-tail potential drop distributed over $40 R_E$) but certainly suggests that a significant reverse electric field can lie along an X line in the distant tail as a result of an induction field dominating the potential field. Note that $|E_y| \propto K/m^{1/4}$ and that $(m_p/m_e)^{1/4} = 6.5$, where K is kinetic energy and subscripts p and e refer to protons and electrons, respectively. Since the range of K_p/K_e in the plasma sheet is about 5–10 [Frank, 1971; Frank et al., 1978], the maximum allowable magnitude for E_y is about the same for protons as it is for electrons in the magnetotail.

3. EVALUATION OF FORCE BALANCE

It is illustrative to evaluate the balance of forces near an X line having $E \cdot J < 0$ as we did in paper 1 for $E \cdot J > 0$. Using the coordinate system and assumptions from section 2 and the normalized variables given by (1) and (2), we obtain the normalized y component of the momentum equation:

$$(1/n')(\nabla' \cdot \mathcal{K}')_y = (-V'_x x' \pm 1) \text{sign}(q), \quad (6)$$

where $n' = L^3 n = \int f(\mathbf{v}') d\mathbf{v}'$ is the normalized density, $\mathcal{K}' = n'(\mathbf{v}'\mathbf{v}')$ is the normalized kinetic tensor, $(\nabla' \cdot \mathcal{K}')_y = (\partial/\partial x')\mathcal{K}'_{xy} + (\partial/\partial z')\mathcal{K}'_{zy}$, and the plus or minus again refers to the sign of E_y . This sign is positive for $E \cdot J > 0$ along the X line and negative for $E \cdot J < 0$. Equation (6) is written so as to be applicable to all species. As in section 2, we take q to be positive, so that the sign of v'_y must be changed to be correct for electrons.

We use trajectories such as those shown in Figure 1 to evaluate normalized distribution functions f' in phase space. Symmetry about $x' = 0$ is assumed, so that for every particle having $x'_0 \geq 0$ and $v'_{x0} \leq 0$ there is a corresponding particle having $x'_0 \leq 0$ and $v'_{x0} \geq 0$. We assume that particles enter the $z = 0$ plane at all x'_0 in the vicinity of the X line. Since such particles convect toward the X line for $E_y < 0$, this latter assumption requires that there be a particle

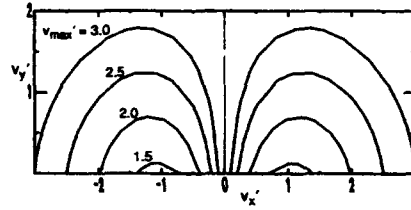


Fig. 2. Boundaries of regions in the v'_x, v'_y plane within which the normalized distribution function $f'(x', \mathbf{v}')$ is nonzero at $x' = 0$ for $v'_{\max} = 1.5, 2, 2.5$, and 3 .

population trapped on field lines that cross the $z = 0$ plane away from the X line.

To illustrate the features of f' , it is convenient to take a "top hat" form for the distribution function of particles f_0 that enter the $z = 0$ plane. We take $f_0(x'_0, v'_0) = \delta(v'_0) f_{20}(v'_{x0}, v'_{z0})$, where $\int f_{20}(v'_{x0}, v'_{z0}) dv'_{z0} = 1$ for $0 \leq -v'_{x0} \text{ sign}(x'_0) \leq v'_{\max}$ and $f_{20} = 0$ for other v'_{x0} . This distribution for particles incident upon the current sheet has a normalized thermal velocity $(V'_{th})^2 = \langle (v'_{x0})^2 \rangle = \frac{1}{2}(v'_{\max})^2$ and a thermal energy $K_{th,0} = m_e [E_y v'_{\max} / L \beta]^2 / 6$. The incident distribution f_0 is taken to be independent of time. Thus, under the assumption that the x, y component and the z component of particle motion are decoupled, $\int f'(x', \mathbf{v}') dv'_z = 1$ along all particle trajectories in the x', y' plane for which f_0 is nonzero.

With the above initial distribution f_0 , the distribution function f' at all x' can be described by an area in v'_x, v'_y space within which $\int f'(x', \mathbf{v}') dv'_z = 1$ and outside of which $f' = 0$. It thus suffices to trace the boundary of this area at any x' in order to describe $f'(x')$. The boundaries in the v'_x, v'_y plane at $x' = 0$ are shown in Figure 2 for $v'_{\max} = 1.5, 2, 2.5$, and 3 . For each v'_{\max} there are two regions having $f'(0, \mathbf{v}') = 1$, one at positive v'_x and one at negative v'_x . The two regions correspond to trajectories started from either side of the X line.

The boundary of each region in Figure 2 is obtained solely from particles initiated with $v'_{x0} = -v'_{\max} \text{sign}(x'_0)$ and with x'_0 within the range that can reach $x' = 0$. The regions also cut off at $v'_y = 0$. The boundaries illustrate the decrease in the range of velocity space within which particles have access at $x' = 0$ as v'_{\max} is decreased. For $v'_{\max} < v'_c (= 1.35)$, no particles reach $x' = 0$.

The variation of the boundaries in velocity space with x' can be seen in Figure 3, where the boundaries at selected x' are shown for $v'_{\max} = 3$. The distributions can be seen to develop a skewing in velocity space with increasing x' . This skewing corresponds to increasingly negative values of $\langle v'_x v'_y \rangle$ near $x' = 0$ that give rise to negative gyroviscosity as is required to balance the electric force for $E \cdot J < 0$.

In order to evaluate terms in the normalized momentum equation, we obtain moments of $f'(x', \mathbf{v}')$ by integrating over the areas enclosed by boundaries such as those shown in Figure 3. We determine the moments $n', V'_x, V'_y, \mathcal{K}'_{xy}$, and P'_{xy} as defined in paper 1 as a function of x' .

If we were to evaluate the particle motion in three dimensions, there would be a nonvanishing value for $(\partial/\partial z')\mathcal{K}'_{zy}$, though $\mathcal{K}'_{zy} = 0$ at $z' = 0$. In our two-dimensional approach to particle motion, this term is replaced by a net source \mathcal{S} of particles having $v'_y = 0$ at $z' = 0$, since there are no

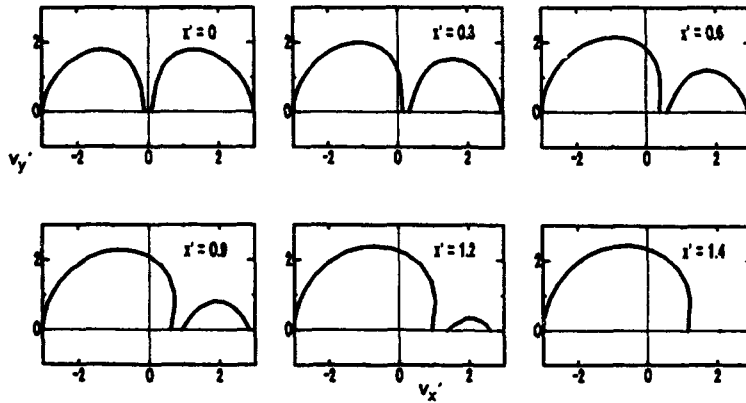


Fig. 3. Boundaries of the region in the v'_x, v'_y plane within which the normalized distribution function $f'(x', v')$ is nonzero at selected values of x' for $v'_{\max} = 3$.

derivatives in the z direction. The net particle source is the sum of a source that maintains $f_0(x_0, v_{x0}, v_{y0})$ at its prescribed value for each v'_{x0} and a sink that removes particles when v'_y returns to 0. In normalized units,

$$\begin{aligned} (\partial/\partial x')\mathcal{K}'_{xy} = (\partial/\partial x')P'_{xy} + V'_y(\partial/\partial x')(n'V'_x) \\ + n'(V'_x\partial/\partial x')V'_y, \end{aligned} \quad (7)$$

where $(\partial/\partial x')(n'V'_x) = \mathcal{S}$. The form of the momentum equation whose terms we evaluate is

$$(1/n')[(\partial/\partial x')\mathcal{K}'_{xy}] + V'_xx' = -1. \quad (8)$$

The results of our evaluation of the moments and the terms of the momentum equation are shown in Figure 4 for $v'_{\max} = 3$. The bottom panel of the figure shows the velocity space moments versus x' . The top panel shows the two terms (thinner solid curves) on the left side of equation (8); their sum (heavier solid curve), which should equal -1 ; and the contributions to $(\partial/\partial x')\mathcal{K}'_{xy}$ from each of the three terms (dotted curves) on the right side of (7).

The top panel of Figure 4 confirms that the force balance specified by equation (8) is maintained within our model. The viscosity term $(1/n')[(\partial/\partial x')\mathcal{K}'_{xy}]$ can be seen to be negative at $x' \leq 1.3$, and the viscosity is the dominant force balancing the electric force at $x' \leq 0.85$. At larger x' the magnetic force becomes increasingly important in relation to the viscous force in balancing the electric force. It can be seen that both $(1/n')[(\partial/\partial x')P'_{xy}]$ and the source from particle transport in the z direction, $(V'_y/n')[(\partial/\partial x')(n'V'_x)]$, are important near $x' = 0$. In a three-dimensional evaluation of particle motion, $(\partial/\partial x')(n'V'_x)$ would equal zero at $x' = 0$, and a term $(1/n')[(\partial/\partial z')P'_{xy}]$ would add to $(1/n')[(\partial/\partial x')P'_{xy}]$ to exactly balance the electric force.

The bottom panel of Figure 4 shows that \mathcal{K}'_{xy} and P'_{xy} decrease from zero as x' increases from zero. This gives the negative viscous force that balances the negative electric force. Neither V'_y nor n' varies significantly with x' for $v'_{\max} = 3$. Thus a significant decrease in current is not expected from the electron acceleration in the negative y' direction that occurs very near the X line. However, as can be seen in Figure 3, both V'_y and n' at $x' = 0$ would decrease if v'_{\max} were decreased. For $v'_{\max} \leq 1.35$, n' would equal zero at $x' = 0$, and no current would flow.

4. SUMMARY AND CONCLUSIONS

This study was motivated by our earlier proposal [Lyons *et al.*, 1989] that plasma could be transferred from closed to open geomagnetic field lines on the nightside of the magnetosphere during periods when the boundary between open and closed field lines moves rapidly toward decreasing Λ . Such transfer requires that $E \cdot J < 0$ along the nightside magnetic

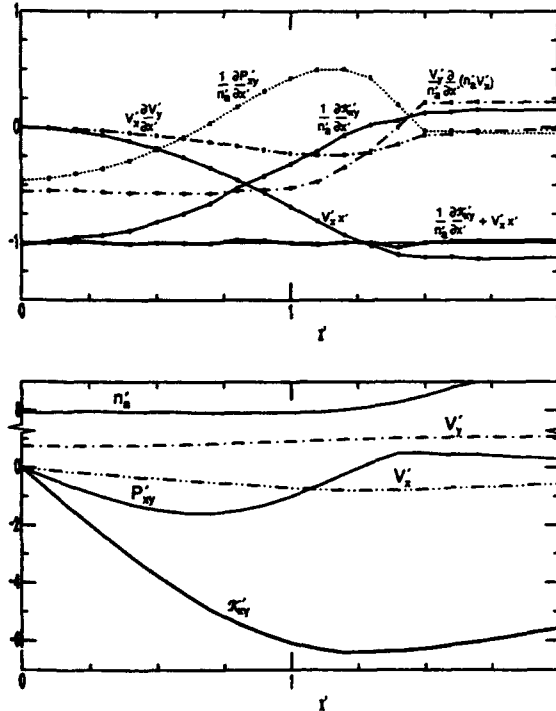


Fig. 4. Moments of $f'(x', v')$ versus x' (bottom panel) and terms in the normalized momentum equation (top panel). In the top panel the two terms on the left-hand side of equation (8) are shown by the thinner solid curves, and their sum, which should equal -1 , is shown by the heavier solid curve. A thin solid line indicates the value -1 . Dotted curves show the contribution to $(1/n')[(\partial/\partial x')\mathcal{K}'_{xy}]$ from each of the three terms on the right-hand side of equation (7).

X line, a condition that is not allowed within the context of resistive reconnection theory. Collisionless reconnection theory, on the other hand, requires a balance between the electric force and a gyroviscous force at an X line and does not a priori require that $\mathbf{E} \cdot \mathbf{J} > 0$. Thus observational evidence that there are periods when $\mathbf{E} \cdot \mathbf{J} < 0$ along the tail X line would suggest that collisionless gyroviscosity, rather than resistivity, is important near an X line. The gyroviscosity is expressed in terms of a gradient of the off-diagonal elements of the electron pressure tensor.

We have investigated conditions for which particle motion within the current sheet in the vicinity of an X line can give a current in the direction appropriate for $\mathbf{E} \cdot \mathbf{J} < 0$. We found that the normalized velocity of a particle entering the current sheet must be above a critical magnitude $v'_c = 1.35$ in order for the particle to be able to carry such a current. This critical velocity was used to obtain a maximum electric field magnitude (given by equation (5)) for which a significant current can be maintained at an X line having $\mathbf{E} \cdot \mathbf{J} < 0$. For parameters typical of the distant magnetotail the critical electric field magnitude was found to be ~ 0.15 mV/m, which is of the order of, though somewhat less than, the potential electric field magnitudes expected in the magnetotail. This maximum allowable field magnitude is about the same for protons as it is for electrons in the magnetotail.

An electric field having $\mathbf{E} \cdot \mathbf{J} < 0$ could only result from induction in the magnetotail and thus at most should not persist for more than a few tens of minutes. Consistent with this suggestion, the radar observations of *de la Beaujardiere et al.* [1991] showed the magnetic separatrix moving equatorward faster than the convecting plasma for a 10- to 15-min period. The magnitude of the electric field in the frame of reference of the moving separatrix, during the period when the electric field was reversed, was somewhat less than the typical magnitudes observed at other times. This is consistent with the maximum magnitude estimated here. These observations are consistent with the proposal that collisionless gyroviscosity, rather than resistivity, provides the force that balances the electric force along an X line.

Our goal has been to evaluate the force balance and conditions for current maintenance in a collisionless plasma having $\mathbf{E} \cdot \mathbf{J} < 0$ in the vicinity of an X line. Making the assumption of steady state ($\nabla \times \mathbf{E} = 0$) has enabled us to apply the normalizations of *Dungey* [1988], which facilitates the scaling of our results to different situations. Equatorward motion of the actual magnetospheric separatrix is associated with an induced electric field, which cannot be uniform as has been assumed here. However, in the vicinity of the tail X line this induced electric field is directed opposite to the direction of the nightside cross-tail current, so that actual conditions should resemble those assumed here. It would be desirable to next investigate particle motion and force bal-

ance in a more realistic field model where the X line motion and $\nabla \times \mathbf{E}$ are consistent with a prescribed $\partial \mathbf{B} / \partial t \neq 0$.

Acknowledgments. We are grateful to J. W. Dungey for stimulating discussions that motivated this work, to M. Schulz for valuable discussions and critical comments on the manuscript, and to two of the referees for encouraging us to place increased emphasis on the reversibility of the particle trajectories. The work was supported by the NASA Space Physics Theory Program grant NAGW-2126 and the Aerospace Sponsored Research Program.

The Editor thanks T. W. Speiser, V. M. Vasyliunas, and another referee for their assistance in evaluating this paper.

REFERENCES

- de la Beaujardiere, O., L. R. Lyons, and E. Friis-Christensen, Sondrestrom radar measurements of the reconnection electric field, *J. Geophys. Res.*, **96**, 13,907, 1991.
- Dungey, J. W., Noise-free neutral sheets, *Eur. Space Agency Spec. Publ.*, ESA SP-285(2), 15, 1988.
- Frank, L. A., Relationship of the plasma sheet, ring current, trapping boundary, and plasmopause near the magnetic equator and local midnight, *J. Geophys. Res.*, **76**, 2265, 1971.
- Frank, L. A., K. L. Ackerson, R. J. DeCoster, and B. G. Burek, Three-dimensional plasma measurements within the Earth's magnetosphere, *Space Sci. Rev.*, **22**, 739, 1978.
- Lyons, L. R., and D. C. Pridmore-Brown, Force balance near an X line in a collisionless plasma, *J. Geophys. Res.*, **95**, 20,903, 1990.
- Lyons, L. R., M. Schulz, and J. F. Fennell, Trapped-particle evacuation: Source of magnetotail bursts and tailward flows?, *Geophys. Res. Lett.*, **16**, 353, 1989.
- Martin, R. F., Jr., and T. W. Speiser, A predicted energetic ion signature of a neutral line in the geomagnetic tail, *J. Geophys. Res.*, **93**, 11,521, 1988.
- Sonnerup, B. U. Ö., Magnetic field reconnection, in *Solar System Plasma Physics III*, edited by L. J. Lanzerotti, C. F. Kennel, and E. N. Parker, p. 45, North-Holland, Amsterdam, 1979.
- Sonnerup, B. U. Ö., On the theory of steady-state reconnection, *Comput. Phys. Commun.*, **49**, 143, 1988.
- Speiser, T. W., Particle trajectories in model current sheets, 1, Analytical solutions, *J. Geophys. Res.*, **70**, 4219, 1965.
- Speiser, T. W., On the uncoupling of parallel and perpendicular particle motion in a neutral sheet, *J. Geophys. Res.*, **73**, 1112, 1968.
- Speiser, T. W., Conductivity without collisions or noise, *Planet. Space Sci.*, **18**, 613, 1970.
- Tidman, D. A., and N. A. Krall, *Shock Waves in Collisionless Plasmas*, pp. 9, 10, 70, Wiley-Interscience, New York, 1971.
- Vasyliunas, V. M., Theoretical models of magnetic field line merging, 1, *Rev. Geophys.*, **13**, 303, 1975.
- Vasyliunas, V. M., Steady state aspects of magnetic field merging, in *Magnetic Reconnection in Space and Laboratory Plasmas*, *Geophys. Monogr. Ser.*, vol. 30, edited by E. W. Hones, Jr., p. 25, AGU, Washington, D. C., 1984.
- Lyons, L. R. and D. C. Pridmore-Brown, Space Sciences Laboratory, M2-260, Aerospace Corporation, P. O. Box 92957, Los Angeles, CA 90009.

(Received March 28, 1991;
revised September 24, 1991;
accepted October 29, 1991.)